Algebra II

Prepared by:
Jessica Lee

Superintendent of Schools:
Marie C. Cirasella, Ed.D.
Director of Curriculum, Instruction, & Assessment:
Melissa Quackenbush

Approved by the Midland Park Board of Education on
July 11, 2017

HS Algebra II Curriculum Overview

High school Algebra II is taught in seven units throughout the school year. This course is designed to be an extension into topics that are taught in Algebra I, as well as an introduction to topics that will aid in transitioning to higher-level math courses. Students in Algebra II will become fluent with a variety of different function families and the ways the same concepts can be threaded through each family. While the foundation remains unchanged, students will look more in depth at how the present family of functions that is being studied affects the formulas and equations. The course also focuses on the use of graphing calculator to facilitate the solving of equations and modeling of real-world problems. The curriculum ends with a look into statistical analysis and probability.

Taking part in this course helps students:
1. To foster an appreciation of mathematics.
2. To observe math in the world around them.
3. To meet the New Jersey Student Learning Standards for New Jersey Public Schools.

Suggested Course Sequence*:

Unit 1: Quadratic Functions (25 days)
Unit 2: Polynomial Functions (26 days)
Unit 3: Rational Exponent and Radical Functions (22 days)
Unit 4: Exponential and Logarithmic Functions (22 days)
Unit 5: Rational Functions (21 days)
Unit 6: Probability (18 days)
Unit 7: Data Analysis and Statistics (18 days)

Pre-Requisite: Algebra I

*The number of instructional days is an estimate based on the information available at this time. 1 day equals approximately 48 minutes of seat time. Teachers are strongly encouraged to review the entire unit of study carefully and collaboratively to determine whether adjustments to this estimate need to be made.

Content Area: Algebra 2

Unit Title: Quadratic Functions

Grade Level: 9/11

Unit Summary: Students will work with the family of quadratic functions. The unit will cover working with different forms of the same equation, solving quadratics with a variety of different methods, and comparing graphs and solutions of quadratic equations to that of quadratic inequalities. Students will learn to determine which method is appropriate based on the form of the equation given.

Interdisciplinary Connections:
Science, English, History, Art

21st Century Themes and Skills:
CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP4. Communicate clearly and effectively and with reason.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

Standards (Content and Technology):	
CPI#:	Statement:
NJSLS.N-CN.A.2	Use the relation i^2 = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
NJSLS.A-SSE.B.3a	Factor a quadratic expression to reveal the zeros of the function it defines
NJSLS.F-IF.C.7a	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
NJSLS.A-REI.B.4a	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2$ = q that has the same solutions. Derive the quadratic formula from this form.
NJSLS.A-REI.B.4b	Solve quadratic equations in one variable. b. Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and
NJSLS.N-CN.C.7	Solve quadratic equations with real coefficients that have complex solutions.
2014 New Jersey Core Curriculum Content Standards - Technology	
8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
8.1.12.E.1	Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.
8.2.12.C.4	Explain and identify interdependent systems and their functions.
Unit Essential Question(s):	Unit Enduring Understandings: • Quadratic parent function $(y = x^2)$

- How can we use quadratic functions to model real life phenomena?
- Why do we need the different but equivalent forms of a quadratic function?
- How can we decide that the quadratic function will be the best fit for a real life situation?
- Can the real number system be extended?
- Why do we need complex numbers?
- How do we use the method of completing the square to transform any quadratic expression?
- How can we solve quadratic equations by taking square roots, completing the square, the Quadratic Formula, and factoring?
- How can we solve quadratic inequalities?

- Parabola, axis of symmetry, vertex, max./min.
- Real solutions of equations show the zeros of the functions which are the x-intercepts of the graphs.
- Imaginary numbers $(i = \sqrt{-1};\ i^2 = -1)$
- Complex number set includes all real numbers.
- Discriminant determines number and type of solutions.

Unit Learning Targets/Objectives:
Students will...
- Graph quadratic functions written in standard, vertex, and intercept forms
- Identify axis of symmetry, vertex, maximum and minimum of graphs
- Solve quadratics by graphing, factoring, square roots, completing the square, Quadratic Formula
- Define i
- Perform operations on complex number
- Solve quadratics with complex solutions and zeros
- Analyze the discriminant to determine type and number of solutions
- Graph and solve quadratic inequalities

Formative Assessments:
Homework, On-spot Checking for Understanding, Teacher Feedback

Summative/Benchmark Assessment(s):
Quizzes, Tests

Resources/Materials:
https://njctl.org/courses/math/algebra-ii/

Textbook:
Larson, Ron, Laurie Boswell, Timothy D. Kanold, and Lee Stiff. *Algebra 2*. Orlando, FL: Holt McDougal, 2012. Print.

Modifications:
- Special Education Students
  - Rephrase questions, directions, and explanations
  - Allow extended time on assessments
  - Consult with Case Managers and follow IEP modifications/accommodations
- English Language Learners
  - Allow errors in speaking

- At-Risk Students
  - Consult with Guidance Counselors and follow I&RS procedures/action plans
  - Consult with classroom teacher(s) for specific behavior interventions
  - Provide extended time to complete tasks (on need basis)
- Gifted and Talented Students
  - Provide extension activities

o   Rephrase questions, directions, and explanations

o   Allow extended time on assessments

o   Build on students' intrinsic motivations

Lesson Name/Topic	Lesson Objective(s)	Time frame (day(s) to complete)
Quadratics in Standard Form	-Graphing quadratics in standard form (with axis of symmetry and vertex)	2 days
Quadratics in Vertex and Intercept Forms	-Graphing quadratics in vertex and intercept forms (with axis of symmetry and vertex)	2 days
Factoring Quadratics	-Factoring quadratics completely and solving quadratics by factoring	4 days
Square Roots	-Solving quadratics by square roots and simplifying non-perfect radicals	2 days
Complex Numbers	-Defining the complex number system and solving quadratics with complex solutions	2 days
Completing the Square	-Solving quadratics by completing the square and transforming standard form into vertex form	3 days
Quadratic Formula	-Solving quadratics using the Quadratic Formula. -Calculating the discriminant to determine type and number of solutions	2 days
Quadratic Inequalities	-Graphing quadratic inequalities on a coordinate grid. -Solving quadratic inequalities graphically and algebraically.	3 days

Teacher Notes: 25 total days including assessment days (quizzes, test)

Additional Resources

Content Area: Algebra 2

Unit Title: Polynomial Functions

Grade Level: 9/11

Unit Summary: Students will work with the family of polynomial functions. The unit will cover higher-degree polynomials and their characteristics and relating the factoring patterns of quadratics to polynomials. Students will also apply the Remainder, Factor, and Rational Theorems to polynomials to solve for zeros. Graphing calculators will be used for their CALCULATE and GRAPH features to aid the solving process. The Fundamental Theorem of Algebra will be used to further classify polynomials based on the number and type of solutions.

Interdisciplinary Connections:
Science, English, History, Art

21st Century Themes and Skills:
CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP4. Communicate clearly and effectively and with reason.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

Standards (Content and Technology):

CPI#:	Statement:
NJSLS.N-RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
NJSLS.F-IF.C.7c	Graph functions expressed symbolically and show key feature of the graph, by hand in simple cases and using technology for more complicated cases. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
NJSLS.A-APR.A.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials
NJSLS.A-APR.B.2	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
NJSLS.A-SSE.A.2	Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*
NJSLS.N-CN.C.9 (+)	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
2014 New Jersey Core Curriculum Content Standards - Technology	
8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
8.1.12.E.1	Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.
8.2.12.C.4	Explain and identify interdependent systems and their functions.

Unit Essential Question(s):	Unit Enduring Understandings: • Definition of a polynomial function

• How can polynomial functions be used to model real life problems? • How can properties of linear and quadratic functions be generalized to polynomial functions? • What are some common characteristics of polynomial graphs? • How do we use the factors of a polynomial to solve a division problem? • How do we factor a polynomial? • What is the Fundamental Theorem of Algebra? • How do the characteristics of quadratics apply to polynomials?	• End behavior of polynomial graphs • Polynomial division (Long and Synthetic) • Algebraic properties of polynomial functions • Degree of a polynomial tells how many roots it has (including repeated and imaginary)

Unit Learning Targets/Objectives:
Students will...
• Identify characteristics of polynomials from the function as well as the graph
• Graph polynomials functions by hand as well as with a graphing calculator
• Identify end behavior of polynomials
• Use the structure of a polynomial expression to identify ways to rewrite the expression using factoring patterns
• Solve polynomials by factoring
• Divide polynomials using long division and synthetic division
• Apply the Remainder and Factor to completely factor polynomials and solve
• Apply the Rational Zero Theorem to create a list of possible rational zeros and solve for rational roots
• Analyze graphs of polynomial functions using the Fundamental Theorem of Algebra

Formative Assessments:
Homework, On-spot Checking for Understanding, Teacher Feedback

Summative/Benchmark Assessment(s):
Quizzes, Tests

Resources/Materials:
https://njctl.org/courses/math/algebra-ii/

Textbook:
Larson, Ron, Laurie Boswell, Timothy D. Kanold, and Lee Stiff. *Algebra 2*. Orlando, FL: Holt McDougal, 2012. Print.

Modifications:
• Special Education Students
  o Rephrase questions, directions, and explanations
  o Allow extended time on assessments
  o Consult with Case Managers and follow IEP modifications/accommodations
• English Language Learners
  o Allow errors in speaking
  o Rephrase questions, directions, and explanations

• At-Risk Students
  o Consult with Guidance Counselors and follow I&RS procedures/action plans
  o Consult with classroom teacher(s) for specific behavior interventions
  o Provide extended time to complete tasks (on need basis)
• Gifted and Talented Students
  o Provide extension activities

o Allow extended time on assessments		o Build on students' intrinsic motivations

Lesson Name/Topic	Lesson Objective(s)	Time frame (day(s) to complete)
Properties of Exponents	-Simplifying algebraic expressions using the properties of exponents -Applying the properties of exponents to scientific notation	2 days
Polynomial Functions	-Identifying the characteristics of polynomials (including their degree, leading coefficient, and constant) -Exploring similarities and differences between even and odd degree polynomials -Graphing polynomials	2 days
Add, Subtract, Multiply Polynomials	-Performing addition, subtraction, and multiplication on sets of polynomials to simplify	2 days
Factoring Polynomials	-Factoring polynomials completely and solving polynomials by factoring	3 days
The Factor and Remainder Theorems	-Dividing polynomials using long division and synthetic division -Using the Remainder Theorem to determine function evaluation -Using the Factor Theorem to identify factors of each polynomial, and subsequently, their zeros	3 days
The Rational Zero Theorem	-Using the Rational Zero Theorem to create a list of possible rational zeros -Applying the Factor Theorem to solve polynomials for their rational zeros	3 days
The Fundamental Theorem of Algebra	-Applying the Fundamental Theorem of Algebra to obtain all possible zeros for a polynomial -Using Descartes's Rule of Signs to determine the number of positive and negative zeros for each polynomial	3 days
Analyzing Graphs of Polynomials	-Exploring relationships between zeros, x-intercepts, factors, and solutions of polynomials	2 days

Teacher Notes: 26 total days including assessment days (quizzes, test)

Additional Resources

Content Area: Algebra 2	
Unit Title: Rational Exponents and Radical Functions	
Grade Level: 9/11	

Unit Summary: Students will work with the family of rational exponent and radical functions. The unit will cover the relationship between rational exponents and radicals, and will extend to the creation of new functions using the function operations and composition. Students will work with graphing square root and cube root functions as well as explore translations of the graphs. The concept of extraneous solutions will be covered when rational exponent and radical equations are solved.

Interdisciplinary Connections:
Science, English, History, Art

21st Century Themes and Skills:
CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP4. Communicate clearly and effectively and with reason.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

Standards (Content and Technology):

CPI#:	Statement:
NJSLS.N-RN.A.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*
NJSLS.N-RN.A.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
NJSLS.F-BF.A.1b	Write a function that describes a relationship between two quantities. b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
NJSLS.F-BF.A.1c	Write a function that describes a relationship between two quantities. c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*

NJSLS.F-BF.B.4	Find inverse functions.
	    a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x^3 or f(x) = (x+1)/(x−1) for x ≠ 1.*
	    b. (+) Verify by composition that one function is the inverse of another.
	    c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
	    d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

NJSLS.F-IF.C.7b	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

	b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
NJSLS.A-REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

2014 New Jersey Core Curriculum Content Standards - Technology

8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
8.1.12.E.1	Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.
8.2.12.C.4	Explain and identify interdependent systems and their functions.

Unit Essential Question(s):	Unit Enduring Understandings:
- How can we use a rational exponent to represent a power involving a radical? - How can we use the properties of exponents to simplify products and quotients of radicals? - How can we find the inverse of radical functions? - How can we solve radical equations? - How are rational exponent and radical functions and their graphs similar to linear functions? How are they different? - How can we identify the domain and range of radical functions?	- Rational exponents are equal to n^{th} roots. - Properties of exponents - Extraneous solutions - Inverses (Not all inverses are functions.) - Undefined fractions (Denominator cannot be zero.) - Fraction operations (Addition, Subtraction, Multiplication, Division) - Operations with polynomials

Unit Learning Targets/Objectives:

Students will...

- Find n^{th} roots of a number
- Evaluate expressions with rational exponents
- Simplify rational expressions using the properties of exponents
- Simplify radical expressions using the properties of radicals (including rationalizing the denominator)
- Graph square root and cube root functions and identify the respective domain and range
- Perform addition, subtraction, multiplication, and division on functions (including composition)
- Explore inverses of rational exponents, radical functions, and power functions
- Solve radical equations

Formative Assessments:
Homework, On-spot Checking for Understanding, Teacher Feedback

Summative/Benchmark Assessment(s):
Quizzes, Tests

Resources/Materials:
https://njctl.org/courses/math/algebra-ii/

Textbook:
Larson, Ron, Laurie Boswell, Timothy D. Kanold, and Lee Stiff. *Algebra 2*. Orlando, FL: Holt McDougal, 2012. Print.

Modifications:
- Special Education Students
- At-Risk Students

- o Rephrase questions, directions, and explanations
- o Allow extended time on assessments
- o Consult with Case Managers and follow IEP modifications/accommodations
- English Language Learners
  - o Allow errors in speaking
  - o Rephrase questions, directions, and explanations
  - o Allow extended time on assessments

- o Consult with Guidance Counselors and follow I&RS procedures/action plans
- o Consult with classroom teacher(s) for specific behavior interventions
- o Provide extended time to complete tasks (on need basis)
- Gifted and Talented Students
  - o Provide extension activities
  - o Build on students' intrinsic motivations

Lesson Name/Topic	Lesson Objective(s)	Time frame (day(s) to complete)
n^{th} roots and Rational Exponents	-Converting between rational exponents and radicals -Evaluating rational exponents and radicals	3 days
Properties of Rational Exponents	-Simplifying rational exponent and radical expressions -Rationalizing the denominator for radical expressions	3 days
Function Operations and Composite Functions	-Creating new functions by adding, subtracting, multiplying and dividing sets of functions -Composing new function	3 days
Inverse Functions	-Finding the inverse of a function by switching the domain and range -Verifying inverses by using the composite functions	3 days
Square Root and Cube Root Functions	-Graphing square root and cube root functions -Identifying domain and range of each function	2 days
Radical Equations	-Solving rational exponent equations -Solving radical equations -Checking for extraneous solutions	3 days

Teacher Notes: 22 total days including assessment days (quizzes, test)

Additional Resources

Content Area: Algebra 2

Unit Title: Exponential and Logarithmic Functions

Grade Level: 9/11

Unit Summary: Students will work with the family of exponential and logarithmic functions. The unit will cover the inverse relationship between exponential and logarithmic functions. The properties of exponents are extended into the properties logarithms which will be used to condense and expand logarithmic expressions. Sets of data can be represented as either exponential or power functions. Students will determine whether an exponential or power function is more appropriate before writing the functions.

Interdisciplinary Connections:

Science, English, History, Art

21st Century Themes and Skills:

CRP1. Act as a responsible and contributing citizen and employee.

CRP2. Apply appropriate academic and technical skills.

CRP4. Communicate clearly and effectively and with reason.

CRP6. Demonstrate creativity and innovation.

CRP7. Employ valid and reliable research strategies

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP9. Model integrity, ethical leadership and effective management.

CRP11. Use technology to enhance productivity.

CRP12. Work productively in teams while using cultural global competence.

Standards (Content and Technology):

CPI#:	Statement:
NJSLS.F-IF.C.7e	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
NJSLS.F-BF.B.5 (+)	Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.
NJSLS.F-LE.A.4	Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
NJSLS.F-LE.A.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
2014 New Jersey Core Curriculum Content Standards - Technology	
8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
8.1.12.E.1	Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.
8.2.12.C.4	Explain and identify interdependent systems and their functions.

Unit Essential Question(s):	Unit Enduring Understandings:
- How can exponential functions be used to model real life problems? - What are some characteristics of exponential and logarithmic functions?	- Exponential form <-> Logarithmic form - Exponential and logarithmic graph translations - Natural base e and natural logarithm ln - Logarithm evaluations

• What is the relationship between exponential and logarithmic functions? • What is the natural base? • How can the properties of exponents be used to derive the properties of logarithms? • How can we solve exponential and logarithmic equations? • How do we determine whether a set of data fits an exponential pattern or a power pattern?	• Logarithm graphs • Exponential growth and decay models • Compound interest and continuously compounded interest models • Abstract and quantitative reasoning

Unit Learning Targets/Objectives:

Students will...

- Graph exponential growth and decay functions and identify domain and range
- Define and use natural base e
- Define and evaluate logarithms
- Apply inverse properties of logarithms and exponential functions
- Graph natural base and logarithmic functions
- Use the properties of logarithms to expand and condense logarithms
- Solve logarithmic and exponential equations
- Write exponential functions modeling real-life problems
- Solve exponential application problems involving growth, decay, and interest

$$< y = a(1+r)^t; \ y = a(1-r)^t; \ A = P(1 + \tfrac{r}{n})^{nt}; \ A = Pe^{rt} >$$

Formative Assessments:

Homework, On-spot Checking for Understanding, Teacher Feedback

Summative/Benchmark Assessment(s):

Quizzes, Tests

Resources/Materials:

https://njctl.org/courses/math/algebra-ii/

Textbook:

Larson, Ron, Laurie Boswell, Timothy D. Kanold, and Lee Stiff. *Algebra 2*. Orlando, FL: Holt McDougal, 2012. Print.

Modifications:

- Special Education Students
    - Rephrase questions, directions, and explanations
    - Allow extended time on assessments
    - Consult with Case Managers and follow IEP modifications/accommodations
- English Language Learners
    - Allow errors in speaking
    - Rephrase questions, directions, and explanations
    - Allow extended time on assessments

- At-Risk Students
    - Consult with Guidance Counselors and follow I&RS procedures/action plans
    - Consult with classroom teacher(s) for specific behavior interventions
    - Provide extended time to complete tasks (on need basis)
- Gifted and Talented Students
    - Provide extension activities
    - Build on students' intrinsic motivations

Midland Park Public Schools

Lesson Name/Topic	Lesson Objective(s)	Time frame (day(s) to complete)
Exponential Growth and Decay	-Graphing exponential growth and decay functions -Applying the growth and decay models to real-life problems -Applying the compound interest formula to real-life problems	3 days
Natural Base *e*	-Simplifying natural base expressions -Evaluating natural base expressions -Identifying growth and decay with natural base exponential functions -Graphing natural base exponential functions	2 days
Logarithms	-Converting between exponential and logarithmic forms -Evaluating logarithms with and without a calculator -Finding inverses of logarithmic functions -Graphing logarithmic functions	3 days
Properties of Logarithms	-Condensing expressions using the properties of logarithms -Expanding expressions using the properties of logarithms	3 days
Exponential and Logarithmic Equations	-Solving exponential equations -Solving logarithmic equations -Checking for extraneous solutions	3 days
Exponential and Power Functions	-Checking whether sets of data fit an exponential or power function -Writing exponential and power functions given a set of points	3 days

Teacher Notes: 22 total days including assessment days (quizzes, test)

Additional Resources

Midland Park Public Schools

Content Area: Algebra 2

Unit Title: Rational Functions

Grade Level: 9/11

Unit Summary: Students will work with the family of rational functions. Rational functions are introduced using inverse and joint variation. The unit will cover the relationship between the graph of rational functions and their characteristics (domain, range, holes, asymptotes). Students will also explore how fraction operations are extended to add, subtract, multiply, and divide rational functions. Rational equations will be solved using skills from Algebra 1 (cross-products, and LCD).

Interdisciplinary Connections:
Science, English, History, Art

21st Century Themes and Skills:
CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP4. Communicate clearly and effectively and with reason.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

Standards (Content and Technology):

CPI#:	Statement:
NJSLS.A-CED.A.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
NJSLS.F-IF.C.7d (+)	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
NJSLS.A-APR.D.7 (+)	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
NJSLS.A-REI.A.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
NJSLS.F-IF.C.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*
2014 New Jersey Core Curriculum Content Standards - Technology	
8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
8.1.12.E.1	Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.
8.2.12.C.4	Explain and identify interdependent systems and their functions.

Unit Essential Question(s):	Unit Enduring Understandings:
• How can rational functions be used to model real-life problems?	• Direct variation vs. Inverse variation • Fraction operations (addition, subtraction, multiplication, division)

• How are inverse variation and rational functions related? • What do vertical/horizontal asymptotes of rational functions signify? • How do we determine excluded values in a rational function? • How can a rational function be solved? • How are rational functions graphed? • How are the four basic operations applied to rational functions? • How do we compare the different characteristics of rational functions?	• Factoring polynomials • Local and global behaviors of rational functions • Translations of functions ($y = \frac{a}{x-h} + k$) • Domain, range, holes, asymptotes

Unit Learning Targets/Objectives:
Students will...
- Graph parent function ($y = \frac{1}{x}$) and their translations ($y = \frac{a}{x-h} + k$)
- Identify domain, range, holes (removable discontinuities), asymptotes
- Simplify rational expressions
- Add and subtract rational expressions
- Multiply and divide rational expressions
- Solve rational equations
- Solve direct, inverse, and joint variation problems
- Compare and contrast function characteristics
- Find inverses of rational functions
- Use rational functions to model real-life application problems

Formative Assessments:
Homework, On-spot Checking for Understanding, Teacher Feedback

Summative/Benchmark Assessment(s):
Quizzes, Tests

Resources/Materials:
https://njctl.org/courses/math/algebra-ii/

Textbook:
Larson, Ron, Laurie Boswell, Timothy D. Kanold, and Lee Stiff. *Algebra 2*. Orlando, FL: Holt McDougal, 2012. Print.

Modifications:
- Special Education Students
    - Rephrase questions, directions, and explanations
    - Allow extended time on assessments
    - Consult with Case Managers and follow IEP modifications/accommodations
- English Language Learners
    - Allow errors in speaking

- At-Risk Students
    - Consult with Guidance Counselors and follow I&RS procedures/action plans
    - Consult with classroom teacher(s) for specific behavior interventions
    - Provide extended time to complete tasks (on need basis)
- Gifted and Talented Students
    - Provide extension activities

Midland Park Public Schools

o Rephrase questions, directions, and explanations		o Build on students' intrinsic motivations
o Allow extended time on assessments		

Lesson Name/Topic	Lesson Objective(s)	Time frame (day(s) to complete)
Inverse and Joint Variation	-Solve direct, inverse, joint variation problems -Use the comparison of two or more units to determine inverse and joint variation	2 days
Rational Functions Graphs	-Graph simple rational functions -Graph translations of rational functions -Identify domain, range, holes, and asymptotes	3 days
Multiply and Divide Rational Functions	-Apply fraction operations to simplify rational expression multiplication and division	3 days
Add and Subtract Rational Functions	-Apply fraction operations to simplify rational expression addition and subtraction	3 days
Rational Equations	-Solve rational equations using cross-products and LCD -CHeck for extraneous solutions	3 days
Function Characteristics	-Determine whether a function is increasing or decreasing over an interval -Determine whether functions are even or odd -Compare functions in different representations	2 days

Teacher Notes: 21 total days including assessment days (quizzes, test)

Additional Resources

Content Area: Algebra 2	
Unit Title: Probability	
Grade Level: 9/11	

Unit Summary: Students will work on probability and odds of simple events. The unit will cover the differences between mutually exclusive and inclusive events, and independent and dependent events. Students will calculate the probabilities of events, using the Addition Rule or the Multiplication Rule depending on the type of event. Throughout the unit, students will work on finding permutations and combinations, a large part in calculating probability where both are a measure of finding groups of r objects out of n. With permutations, the order in which objects are picked determine a different outcome. With combinations, the order in which objects are picked do not matter.

Interdisciplinary Connections:
Science, English, History, Art

21st Century Themes and Skills:
CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP4. Communicate clearly and effectively and with reason.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

Standards (Content and Technology):

CPI#:	Statement:
NJSLS.S-CP.A.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements, of other events ("or," "and," "not")
NJSLS.S-CP.B.9 (+)	Use permutation and combinations to compute probabilities of compound events and solve problems
NJSLS.S-CP.A.2	Understand that two events A and B are independent if the probability of A and B occurring is the product of their probabilities, and use this characterization to determine if they are independent
NJSLS.S-CP.A.3	Understand that the conditional probability of A given B as $P(A \text{ and } B) / P(B)$, and interpret independence of A and B saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B
NJSLS.S-CP.B.7	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model
NJSLS.S-CP.B.8 (+)	Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A) * P(B|A) = P(B) * P(A|B)$ and interpret the answer in terms of the model
2014 New Jersey Core Curriculum Content Standards - Technology	
8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
8.1.12.E.1	Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.
8.2.12.C.4	Explain and identify interdependent systems and their functions.
Unit Essential Question(s):	**Unit Enduring Understandings:**

- How can we list the possible outcomes in the sample space of an experiment?
- How can we determine whether two events are independent or dependent?
- How can we find probabilities of disjoint and overlapping events?
- How can a tree diagram help us visualize the number of ways in which two or more events can occur?
- How can we determine the frequency of each outcome of an event?

- Counting Principle of Multiplication
- Permutations
- Combinations
- Mutually exclusive events vs. Inclusive events
- Independent events vs. Dependent events
- Complementary events

Unit Learning Targets/Objectives:
Students will...
- Define a sample space for each event
- Find probability using permutations
- Find probability using combinations
- Calculate the theoretical probability of events
- Find probabilities of disjoint and overlapping events
- Find probabilities of the complement of an event
- Find probabilities of independent and dependent events

Formative Assessments:
Homework, On-spot Checking for Understanding, Teacher Feedback

Summative/Benchmark Assessment(s):
Quizzes, Tests

Resources/Materials:
https://njctl.org/courses/math/algebra-ii/

Textbook:
Larson, Ron, Laurie Boswell, Timothy D. Kanold, and Lee Stiff. *Algebra 2*. Orlando, FL: Holt McDougal, 2012. Print.

Modifications:
- Special Education Students
  - Rephrase questions, directions, and explanations
  - Allow extended time on assessments
  - Consult with Case Managers and follow IEP modifications/accommodations
- English Language Learners
  - Allow errors in speaking
  - Rephrase questions, directions, and explanations
  - Allow extended time on assessments

- At-Risk Students
  - Consult with Guidance Counselors and follow I&RS procedures/action plans
  - Consult with classroom teacher(s) for specific behavior interventions
  - Provide extended time to complete tasks (on need basis)
- Gifted and Talented Students
  - Provide extension activities
  - Build on students' intrinsic motivations

Lesson Name/Topic	Lesson Objective(s)	Time frame (day(s) to complete)

Probabilities and Odds	-Analyze the difference between probabilities and odds -Calculate simple probability	2 days
Probabilities using Permutations	-Apply the permutation formula (used when order matters) -Find the probabilities of events involving permutations	3 days
Probabilities using Combinations	-Apply the combination formula (used when order does not matter) -Find the probabilities of events involving similar tirangles	3 days
Disjoint and Overlapping Events	-Determine the difference between disjoint ("or") and overlapping ("and") <mutually exclusive vs. inclusive> -Calculate probabilities of disjoint and overlapping events	3 days
Independent and Dependent Events	-Determine the difference between independent and dependent -Calculate probabilities of independent and dependent events	3 days
Teacher Notes: 18 total days including assessment days (quizzes, test) **Additional Resources**		

Content Area: Algebra 2

Unit Title: Data Analysis and Statistics

Grade Level: 9/11

Unit Summary: Students will explore the world of statistics through the eyes of a researcher. The unit will cover a review of using combinations to find probability and then move onto various statistical data sets. With each data set, students will determine type of distribution and calculate the measures of central tendency and variation accordingly. Students will also design their own experimental and observational studies, collect data, and draw conclusions from their data, taking into account, any possible bias or margins of error.

Interdisciplinary Connections:

Science, English, History, Art

21st Century Themes and Skills:

CRP1. Act as a responsible and contributing citizen and employee.

CRP2. Apply appropriate academic and technical skills.

CRP4. Communicate clearly and effectively and with reason.

CRP6. Demonstrate creativity and innovation.

CRP7. Employ valid and reliable research strategies

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP9. Model integrity, ethical leadership and effective management.

CRP11. Use technology to enhance productivity.

CRP12. Work productively in teams while using cultural global competence.

Standards (Content and Technology):

CPI#:	Statement:
NJSLS.A-APR.C.5 (+)	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
NJSLS.S-MD.A.3 (+)	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
NJSLS.S-ID.A.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
NJSLS.S-IC.A.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
NJSLS.S-IC.B.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
2014 New Jersey Core Curriculum Content Standards - Technology	
8.1.12.C.1	Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
8.1.12.E.1	Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.
8.2.12.C.4	Explain and identify interdependent systems and their functions.

Unit Essential Question(s):	Unit Enduring Understandings:
	• Combinations and permutations

• In a normal distribution, what percent of data lies within k standard deviations of the mean?	• Sample space of an event
• How can we test theoretical probability?	• Probability
• What are some considerations when undertaking a statistical study?	• Recognize data sets that are normal
• How can we test a hypothesis about an experiment?	• Analyze hypotheses and methods of collecting data
• How can we collect data to test a conjecture or draw a conclusion?	• Different sampling methods (random, self-selected, systematic, convenience)
• How can we collect data that accurately represents a population?	• Bias in sampling and survey questions
• What is a binomial distribution?	• z-scores
• How is a binomial distribution related to Pascal's Triangle?	• Pascal's Triangle
	• Measures of Central Tendency and Variation

Unit Learning Targets/Objectives:
Students will...
- Apply the Binomial Theorem to binomial expansion
- Use Pascal's Triangle to determine combinations of binomial distributions
- Use the Empirical Rule to determine normality
- Use z-scores to find probabilities and compare data
- Analyze measures of central tendency (mean, median, mode)
- Analyze measures of variation (variance, standard deviation)
- Estimate and interpret areas under a normal curve
- Analyze hypotheses using simulations
- Recognize bias in survey questions and sampling
- Recognize how randomization applies to experiments and observational studies
- Analyze experimental designs

Formative Assessments:
Homework, On-spot Checking for Understanding, Teacher Feedback

Summative/Benchmark Assessment(s):
Quizzes, Tests

Resources/Materials:
https://njctl.org/courses/math/algebra-ii/

Textbook:
Larson, Ron, Laurie Boswell, Timothy D. Kanold, and Lee Stiff. *Algebra 2*. Orlando, FL: Holt McDougal, 2012. Print.

Modifications:
- Special Education Students
  - Rephrase questions, directions, and explanations
  - Allow extended time on assessments
  - Consult with Case Managers and follow IEP modifications/accommodations
- English Language Learners
  - Allow errors in speaking

- At-Risk Students
  - Consult with Guidance Counselors and follow I&RS procedures/action plans
  - Consult with classroom teacher(s) for specific behavior interventions
  - Provide extended time to complete tasks (on need basis)
- Gifted and Talented Students

o Rephrase questions, directions, and explanations	o Provide extension activities
o Allow extended time on assessments	o Build on students' intrinsic motivations

Lesson Name/Topic	Lesson Objective(s)	Time frame (day(s) to complete)
Binomial Theorem	-Use combinations to determine probabilities -Use Pascal's Triangle to find the number of combinations -Apply the Binomial Theorem to binomial expansions	3 days
Binomial Distributions	-Construct a probability distribution -Determine whether a probability distribution can be classified as a binomial distribution -Calculate probability of a specific number of success in a binomial distribution	3 days
Normal Distributions	-Use the Empirical Rule to determine normality -Calculate area under a normal curve -Calculate the z-score for a standard normal distribution and use it to find probabilities	3 days
Conclusions from Samples	-Classify samples (subsets) of population -Recognize bias in sampling -Calculate margin of error for a random sample of size n taken from a large population	3 days
Experimental and Observational Studies	-Identify and correct bias in survey questioning -Identify experimental and observational studies -Explore comparative studies and causality -Design experimental and observational studies	3 days
Teacher Notes: 18 total days including assessment days (quizzes, test) Additional Resources		